Homework 1 and Study Problems - MATH 225

In this document, you will find two types of problems: homework and study problems. You are required to submit **only the homework problems** to Gradescope. The study problems are intended to help you grasp the topics thoroughly and prepare for exams. It is strongly advised to attempt all study problems for a comprehensive understanding.

Please submit your homework to Gradescope until **January 21, 11pm.**

Homework problems

1. Let $A = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}_{5\times 5}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 4 \\ 2 & 0 & 0 & 1 \end{bmatrix}_{5\times 4}$.

   (a) What is the second column of the product AB?

   (b) What is the third row of the product AB?

   (c) What is the entry at fifth row and fourth column of the product AB?

   Note that you don't need to compute all the product to answer these.

2. If A and B are $n \times n$ matrices, prove that $tr(AB) = tr(BA)$. Also give an example of 3×3 matrices A and B such that $AB \neq BA$ but observe via computation $tr(AB) = tr(BA)$ (I mean don't use the proof, compute the traces).

3. Construct *distinct* 2×3 matrix functions A and B defined on all of $\mathbb{R}$ (real numbers) such that $A(0) = B(0)$ and $A(1) = B(1)$. In other words, find two matrix functions $A(t)$ and $B(t)$ with dimension 2×3 such that $A \neq B$ but they agree on values 0 and 1.

4. If A is an $n \times n$ matrix, then the matrices B and C defined by

   $$B = \frac{1}{2}(A + A^T), \quad C = \frac{1}{2}(A - A^T).$$

   B is called *symmetric part* of A, and C is called *anti-symmetric part* of A.

   (a) Use the properties of transpose to show that B is symmetric and C is anti-symmetric.

   (b) Show that $A = B + C$. (This means that we can write any square matrix as the sum of a symmetric matrix and and anti-symmetric matrix.)

5. Suppose A is an $m \times n$ matrix and C is an $r \times s$ matrix.

   (a) Find the dimensions of a matrix B be in order for the product ABC to be defined?

   (b) Write an expression for ij entry of ABC in terms of the entries of A, B, and C.

Study problems

1. To become familiar with matrix algebra, try solving exercises from 2.2.1 to 2.2.4 in the textbook.
2. Enhance your knowledge about matrices by reviewing True-False parts of sections 2.1 (Page 120) and 2.2 (Page 134).
3. Problems 2.1.32, 2.1.33, 2.2.31, 2.2.37, 2.2.38, 2.2.39, and propositions in lecture notes are good for exercising proofs.

   Note that for problems 2.2.37 and 2.2.38, the notation $diag(d_1, d_2, \cdots, d_n)$ stands for $n \times n$ diagonal matrix such that d_i is the diagonal entry at ii-th position. E.g.

   $$diag(3, 4, 5) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

4. For more info about matrix functions and its algebra/calculus, you can read pages 132,133.