# Homework 2 and Study Problems - MATH 225

In this document, you will find two types of problems: homework and study problems. You are required to submit **only the homework problems** to Gradescope. The study problems are intended to help you grasp the topics thoroughly and prepare for exams. It is strongly advised to attempt all study problems for a comprehensive understanding.

Please submit your homework to Gradescope until **January 28, 11pm.**

## Homework problems

1. Reduce the given matrix to reduced row-echelon form and hence determine the rank of each matrix. It is required to specify all row operations you applied. (Note that in order to calculate the rank, reducing to row-echelon form is enough.)

$$a)\begin{bmatrix}3 & 7 & 10\\ 2 & 3 & -1\\ 1 & 2 & 1\end{bmatrix} \qquad b)\begin{bmatrix}1 & -1 & -1 & 2\\ 3 & -2 & 0 & 7\\ 2 & -1 & 2 & 4\\ 4 & -2 & 3 & 8\end{bmatrix}$$

2. Determine the solution set to the system $A\mathbf{x} = \mathbf{b}$ for the given coefficient matrix $A$ and the constant vector $\mathbf{b}$:

$$A = \begin{bmatrix}1 & -1 & 0 & -1\\ 2 & 1 & 3 & 7\\ 3 & -2 & 1 & 0\end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix}2\\ 2\\ 4\end{bmatrix}.$$

3. Determine the solution set to the system $A\mathbf{x} = \mathbf{0}$ for the given matrix $A$:

$$a)A = \begin{bmatrix}1 & 0 & 3\\ 3 & -1 & 7\\ 2 & 1 & 8\\ 1 & 1 & 5\\ -1 & 1 & -1\end{bmatrix} \qquad b)A = \begin{bmatrix}2+i & i & 3-2i\\ i & 1-i & 4+3i\\ 3-i & 1+i & 1+5i\end{bmatrix}$$

4. Determine all values of the constant $k$ for which the following system has **(a)** no solution, **(b)** an infinite number of solutions, and **(c)** a unique solution.

$$\begin{aligned} kx + y + z &= 1,\\ x + ky + z &= 1,\\ x + y + kz &= 1. \end{aligned}$$

5. Determine all values of the constants $a$ and $b$ for which the following system has **(a)** no solution, **(b)** an infinite number of solutions, and **(c)** a unique solution.

$$\begin{aligned} x + 2y - z &= 3,\\ 2x - 3y + az &= b,\\ -x + 4y - 2z &= 5. \end{aligned}$$

## Study problems

1. To understand the difference between row-echelon and reduced row-echelon matrices, solve 2.4.1-8 in the textbook.

2. Enhance your knowledge about rank, reduced forms, and eliminations by reviewing True-False parts of sections 2.4 (Page 155) and 2.5 (Page 165).

3. Problems 2.4.9-18, 2.5.1-18, and 2.5.36-46 are highly recommended for practice to become familiar with reduction and Gaussian elimination algorithms.

4. The final two questions of this homework resemble the style of potential exam questions. Therefore, I strongly recommend that you practice these types of problems further. Similar exercises can be found online for additional practice.