Homework 3 and Study Problems - MATH 225

In this document, you will find two types of problems: homework and study problems. You are required to submit **only the homework problems** to Gradescope. The study problems are intended to help you grasp the topics thoroughly and prepare for exams. It is strongly advised to attempt all study problems for a comprehensive understanding.

Please submit your homework to Gradescope until **February 4, 11pm.**

Homework problems

1. Use the inverse matrix approach to find the solution to the given systems:

$$a)\ \begin{aligned} x+3y &= 1 \\ 2x+5y &= 3 \end{aligned} \qquad\qquad b)\ \begin{aligned} x+y-2z &= -2 \\ y+z &= 3 \\ 2x+4y-3z &= 1 \end{aligned}$$

Recall the approach : in order to solve $A\mathbf{x} = \mathbf{b}$, if A is invertible, it is enough to take $\mathbf{x} = A^{-1}\mathbf{b}$. Please don't forget to find the exact solution.

2. Suppose a is a real number. Find the inverse of the matrix

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & a \\ 0 & 0 & 1 \end{bmatrix}.$$

3. Prove the following statements:

   (a) If A, B, C are invertible, then ABC is invertible, and $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

   (b) (Left cancellation law) If A is invertible and $AB = AC$, then $B = C$.

   (c) (Right cancellation law) If A is invertible and $BA = CA$, then $B = C$.

   (d) Let A be $n \times n$ matrix such that $A^k = 0$ for some positive integer k. Then prove that $I_n - A$ is invertible. (Hint: You can find the inverse of $I_n - A$ as we did in the discussion for $k = 4$.)

4. A square matrix A is said to be **idempotent** if $A^2 = A$.

   (a) Show that if A is idempotent, then $I_n - A$ is also idempotent.

   (b) Show that if A is idempotent, then $2A - I_n$ is invertible and its inverse is itself.

5. Recall one of the equivalences in the Inverse Matrix Theorem:

   A is invertible if and only if A can be written as a product of elementary matrices.

   Use this equivalence to prove that if A and B are invertible $n \times n$ matrices, then AB is invertible. (Hint: It means that, in order to show AB is invertible, you should show AB can be written as a product of elementary matrices.)

Study problems

1. True-False Reviews on pages 176,177, 186(a-g), and 189.
2. Problems 2.6.1-2.6.26 are enough to study on inverse calculations.
3. Problems 2.7.1-2.7.15 are enough to study on elementary matrices.
4. Attempt to prove the properties of inverse matrices independently once more, as this will help you become accustomed to writing proofs.