

Homework 5 and Study Problems - MATH 225

In this document, you will find two types of problems: homework and study problems. You are required to submit **only the homework problems** to Gradescope. The study problems are intended to help you grasp the topics thoroughly and prepare for exams. It is strongly advised to attempt all study problems for a comprehensive understanding.

Please submit your homework to Gradescope until **February 25, 11pm**.

Homework problems

1. (a) On \mathbb{R}^2 , define the operations of addition and scalar multiplication as follows:

$$\begin{aligned}(x_1, x_2) \oplus (y_1, y_2) &:= (x_1 - x_2, y_1 - y_2) \\ k \odot (x_1, x_2) &:= (-kx_1, -kx_2)\end{aligned}$$

Which of the conditions for a vector space are satisfied with these operations? Is this a vector space structure?

- (b) On $M_2(\mathbb{R})$, define the operation of addition by

$$A \oplus B := AB,$$

and use the usual scalar multiplication. Determine which conditions for a vector space are satisfied by $M_2(\mathbb{R})$ with these operations.

2. Determine whether given sets S are a subspace of the given vector spaces V

(a) $S = \{(x, y) \mid x^2 - y^2 = 0\}$ and $V = \mathbb{R}^2$.

(b) $S = \{A \in M_n(\mathbb{R}) \mid \text{tr}(A) = 0\}$ and $V = M_n(\mathbb{R})$.

3. Prove that the space of polynomials of degree n or less, namely P_n , is a subspace of the space of real valued functions $\text{Fun}(\mathbb{R}, \mathbb{R})$. Hint: The proof is just one sentence :)

4. Determine whether the given vector \mathbf{v} is an element of $\text{span}\{v_1, v_2\}$.

- $\mathbf{v} = (3, 3, 4)$, $v_1 = (1, -1, 2)$, $v_2 = (2, 1, 3)$.
- $\mathbf{v} = (5, 3, -6)$, $v_1 = (-1, 1, 2)$, $v_2 = (3, 1, -4)$.
- $\mathbf{v} = (1, 1, -2)$, $v_1 = (3, 1, 2)$, $v_2 = (-2, -1, 1)$.

5. Determine a spanning set for the null space of $A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 1 & 3 & 4 & 2 \\ 2 & 4 & 6 & -1 \end{bmatrix}$.

Study problems

1. True-False Reviews on pages 261, 262, 272, and 282.
2. Sections from 4.2 to 4.4 contains lots of exercises, solve them to study vector spaces, subspaces, and spans.
3. 4.2.1-15 for vector space exercises
4. 4.3.1-20 for subspace exercises
5. 4.3.23-29 for null space exercises
6. All exercises in section 4.4.