

Homework 6 and Study Problems - MATH 225

In this document, you will find two types of problems: homework and study problems. You are required to submit **only the homework problems** to Gradescope. The study problems are intended to help you grasp the topics thoroughly and prepare for exams. It is strongly advised to attempt all study problems for a comprehensive understanding.

Please submit your homework to Gradescope until **March 3, 11pm**.

Homework problems

- Determine whether the given set of vectors is linearly dependent or independent in \mathbb{R}^n for given n . In the case of linear dependence, find a dependency relationship.
 - $\{(2, -1), (3, 2), (0, 1)\}$, $n = 2$.
 - $\{(1, 2, 3), (1, -1, 2), (1, -4, 1)\}$, $n = 3$.
 - $\{(1, -1, 2, 3), (2, -1, 2, -1), (-1, 1, 1, 1)\}$, $n = 4$.
- Determine all values of the constant k for which the given vectors $(1, 1, k)$, $(0, 2, k)$, and $(1, k, 6)$ are linearly dependent in \mathbb{R}^3 .
- Determine whether the following statements are true or false. Give explanation for your answers.
 - If a set contains the zero vector, it is linearly dependent.
 - If $\mathbf{z} \in \text{span}\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.
 - If a 2×2 matrix has linearly independent columns, then its columns span \mathbb{R}^2 .
 - If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ and \mathbf{x} is not a multiple of \mathbf{y} , then $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent.
- Determine whether given set of vectors is linearly independent.
 - $A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 2 & 1 \\ 5 & 7 \end{bmatrix}$ in $M_2(\mathbb{R})$.
 - $p_1(x) = 1 - 3x^2$, $p_2(x) = 2x + x^2$, $p_3(x) = 5$ in $P_2(\mathbb{R})$.
- Use the Wronskian to show that given functions are linearly independent on the given interval I .
 - $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = x^2$, $I = (-\infty, \infty)$.
 - $f_1(x) = \sin x$, $f_2(x) = \cos x$, $f_3(x) = \tan x$, $I = (-\frac{\pi}{2}, \frac{\pi}{2})$.

Study problems

1. True-False Reviews on page 296.
2. All exercises in Section 4.5. Especially, I recommend exercises from 1 to 35.
3. Exercises 4.5.46, 48, 49, 50, 53 are good exercises for proof related problems.