Homework 7 and Study Problems - MATH 225

In this document, you will find two types of problems: homework and study problems. You are required to submit **only the homework problems** to Gradescope. The study problems are intended to help you grasp the topics thoroughly and prepare for exams. It is strongly advised to attempt all study problems for a comprehensive understanding.

Please submit your homework to Gradescope until **March 12, 11pm**.

Homework problems

1. Determine whether the given set S of vectors is a basis for $P_n(\mathbb{R})$.
   - $n = 1$, $S = \{2 - 5x, 3x, 7 + x\}$.
   - $n = 2$, $S = \{-2x + x^2, 1 + 2x + 3x^2, 1 - x^2, 5x + 5x^2\}$.
   - $n = 3$, $S = \{1 + x^3, x + x^3, x^2 + x^3\}$.
   - $n = 3$, $S = \{1 + x + 2x^3, 2 + x + 3x^2 - x^3, -1 + x + x^2 - 2x^3, 2 - x + x^2 + 2x^3\}$.

   The hint is not only for the solution, but you can use Wronskian method to determine independency of polynomials (i.e. functions).

2. The set $\mathbb{C}^n = \{(v_1, v_2, \ldots, v_n)\Big| v_i \in \mathbb{C}\}$ is a both a vector space over $\mathbb{R}$ and a vector space over $\mathbb{C}$. The addition and scalar multiplication are defined as usual, but we change the scalar set only.
   (a) What is the dimension of $\mathbb{C}^n$ as a real vector space? Determine a basis.
   (b) What is the dimension of $\mathbb{C}^n$ as a complex vector space? Determine a basis.

3. Find the change-of-basis matrix $P_{C \leftarrow B}$ from the given ordered basis

$$B = \left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

   to the ordered basis

$$C = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

   of the vector space $M_2(\mathbb{R})$.

4. Let $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 1 & -2 & 6 \\ 3 & 1 & 4 & 2 \end{bmatrix}$. Find a basis for $rowspace(A)$ and $colspace(A)$.

5. Show that a 5×7 matrix A must have $2 \leq nullity(A) \leq 7$. Give an example of a 5×7 matrix A with $nullity(A) = 2$ and an example of a 5×7 matrix A with $nullity(A) = 7$.

Study problems

1. True-False Review in Pages 308, 317, 329, 330 from the textbook
2. Exercises from 4.6.1 to 4.6.30 are good for studying on bases.
3. Exercises from 4.7.17 to 4.7.32 are good for studying on change-of-basis matrices.
4. Exercises from 4.8.1 to 4.8.18 are good for studying on row spaces and column spaces.
5. Exercises from 4.9.1 to 4.9.19 are good for studying on rank-nullity.