QUIZ 10 - MATH 225
SOLUTIONS

1. Determine whether given matrix A is diagonalizable or not. Where possible, find the invertible matrix S and the diagonal matrix D that satisfy $S^{-1}AS = D$.

$$(a)A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix} \quad (b)A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}.$$

(a) $\det(A - \lambda I) = \det \begin{pmatrix} 2-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 2 & -1 & 1-\lambda \end{pmatrix} = (2-\lambda)(1-\lambda)^2,$

So we have two eigenvalues: $\lambda_1 = 2$ with algebraic multiplicity 1, $\lambda_2 = 1$ with algebraic multiplicity 2.

For $\lambda_2 = 1$ We need to solve $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$

The system gives $x = 0, y = 0, z =$ free so $v = (0, 0, z)$ for $z \in \mathbb{R}$. Since the eigenspace has dimension 1, we get geometric multiplicity $\neq$ algebraic multiplicity.

So A is not diagonalizable.

(b) $\det(A - \lambda I) = \det \begin{pmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{pmatrix} = (1-\lambda)(2-\lambda) - 6 = \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1).$

Since A has two distinct eigenvalues, A is non-defective and so diagonalizable. For $\lambda = 4$, an eigenvector is $(1, 1)$. For $\lambda = -1$, an eigenvector is $(-\frac{3}{2}, 1)$. So we can take

$S = \begin{pmatrix} 1 & -\frac{3}{2} \\ 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}$. It gives $S^{-1}AS = D$.