

QUIZ 11 - MATH 225

SOLUTIONS

1. Solve the IVP on $(0, \infty)$

$$y' - \frac{1}{x}y = 2x^2,$$
$$y(1) = 5.$$

(Recall that $a(\ln b) = \ln b^a$ and $\int \frac{1}{x}dx = \ln x + C$.)

2. Determine the general solution to the

$$(D - 1)^3(D^2 + 9)y = 0.$$

(Be careful about the complex roots. There might be such roots.)

Solution 1: Integrating factor: $I(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = x^{-1}$.

$$x^{-1}(y' + \frac{1}{x}y) = x^{-1}(-2x^2)$$

$$\Rightarrow (yx^{-1})' = 2x.$$

Integrate both sides and get

$$yx^{-1} = x^2 + C,$$

so $y = x^3 + Cx$. Using $y(1) = 5$, we find $C = 4$. So the solution is

$$y = x^3 + 4x.$$

Solution 2: The roots are 1 (with multiplicity 3) and $\pm 3i$. So the general solution is

$$c_1e^x + c_2xe^x + c_3x^2e^x + c_4\cos(3x) + c_5\sin(3x).$$