QUIZ 13 - MATH 225 - SOLUTION

1. Solve $\mathbf{x}' = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{x}$.

   The characteristic polynomial of the given matrix is $-\lambda^3 + 3\lambda^2$.

   The eigenvalues are 0 and 3.

   For $\lambda = 0$, we have

   $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{v} = \mathbf{0} \Rightarrow \mathbf{v} = \begin{bmatrix} -y - z \\ y \\ z \end{bmatrix} = \begin{bmatrix} -y \\ y \\ 0 \end{bmatrix} + \begin{bmatrix} -z \\ 0 \\ z \end{bmatrix}.$$

   For $\lambda = 3$, we have
   $$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \mathbf{v} = \mathbf{0} \Rightarrow \mathbf{v} = \begin{bmatrix} x \\ x \\ x \end{bmatrix}.$$

   So the general solution is

   $$\mathbf{x} = c_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + c_3 e^{3x} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

2. Write just one information that you did not know before taking the course, but have learned now. Explain it in 2-3 sentences.

   Depends on a student :)