

## QUIZ 3 - MATH 225

### Solutions

1. Let  $A$  be  $2 \times 2$  matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . Prove that if  $ad - bc \neq 0$ , then  $A$  is invertible and the inverse is

$$\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

*Proof.* Since  $ad - bc \neq 0$  we can take its multiplicative inverse. Then check the identity matrix equation, namely,

$$\begin{aligned} & \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ = & \frac{1}{ad - bc} \begin{bmatrix} da - bc & db - bd \\ -ca + ac & -cb + ad \end{bmatrix} \\ = & \begin{bmatrix} \frac{da - bc}{ad - bc} & 0 \\ 0 & \frac{-cb + ad}{ad - bc} \end{bmatrix} \\ = & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

□

2. Determine whether  $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$  is invertible or not. If  $A$  is invertible, then find the inverse  $A^{-1}$ . Hint: Use the previous question.

*Solution.* Since  $3 * 5 - 7 * 2 = 1 \neq 0$ , the matrix is invertible by previous question. The inverse is

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}.$$

3. **Afternoon session** Determine whether  $A = \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$  is invertible or not. If  $A$  is invertible, then find the inverse  $A^{-1}$ . Hint: Use the previous question.

*Solution.* Since  $4 * 5 - 6 * 3 = 2 \neq 0$ , the matrix is invertible by previous question. The inverse is

$$\frac{1}{2} \begin{bmatrix} 5 & -6 \\ -3 & 4 \end{bmatrix}.$$