QUIZ 4 - MATH 225

Solutions

Question 1. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 2 & 1 & 1 \end{bmatrix}$. Find the determinant by cofactor expansion along the second row.

Solution.

$$
\begin{aligned}
\det(A) &= 2C_{21} + C_{22} + 3C_{23} \\
&= -2\det\left(\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\right) + \det\left(\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\right) - 3\det\left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right) \\
&= -2(1) + (-1) - 3(-3) \\
&= 6
\end{aligned}
$$

Question 2. Let $B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 2 & 1 & 3 & 1 \end{bmatrix}$. Compute the determinant by cofactor expansion. Pick the easiest row or column to use.

Solution.

$$
\begin{aligned}
\det(B) &= 2C_{34} \\
&= -2\det\left(\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}\right) \\
&= -2(C_{11}) \\
&= -2\left(\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}\right) \\
&= -4
\end{aligned}
$$