QUIZ 6 - MATH 225

SOLUTIONS

1. Determine whether the given set of vectors is linearly dependent or independent.

   (a) $\{(2,0,4),(1,0,1),(4,3,1)\}$
   Let $x, y, z \in \mathbb{R}$ be such that

$$x(2,0,4)+y(1,0,1)+z(4,3,1)=(0,0,0).$$

   Then we get

$$\begin{aligned} 2x+y+4z &= 0 \\ 3z &= 0 \\ 4x+y+z &= 0 \end{aligned}$$

   You can directly see $z=0$, then $4x+y=2x+y$ from the first and the third equation, so $4x=2x$ which means $x=0$, and hence $y=0$. Thus, the vectors are linearly independent.

   You can also show it via $\det\left(\begin{bmatrix} 2 & 1 & 4 \\ 0 & 0 & 3 \\ 4 & 1 & 2 \end{bmatrix}\right)=6$. Since the determinant of the coefficient matrix is nonzero, the system has only trivial solution, so the vectors are linearly independent.

   (b) $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} -2 & -6 \\ -4 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}\right\}$
   Let $x, y, z, t \in \mathbb{R}$ be such that

$$x\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}+y\begin{bmatrix} -2 & -6 \\ -4 & 2 \end{bmatrix}+z\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}+t\begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

   Then we get

$$\begin{aligned} x-2y+3z &= 0 \\ -6y+z+5t &= 0 \\ -4y+2z+2t &= 0 \\ 4x+2y+2y+t &= 0 \end{aligned}$$

   Since we have $\det\left(\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -6 & 1 & 5 \\ 0 & -4 & 2 & 2 \\ 4 & 2 & 1 & 1 \end{bmatrix}\right)=0$, the system has infinitely many solutions, so a nontrivial solution. This means that the vectors are linearly dependent.

   Also, you can see directly that

$$\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}=\begin{bmatrix} -2 & -6 \\ -4 & 2 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}+\begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}$$

   This proves again the vectors are linearly dependent.