

QUIZ 7 - MATH 225

SOLUTIONS

1. Consider two ordered bases of \mathbb{R}^2 ; $B = \{(9, 2), (4, -3)\}$ and $C = \{(2, 1), (-3, 1)\}$. Find the change-of-basis matrix $P_{C \leftarrow B}$.

We should solve

$$(9, 2) = a(2, 1) + b(-3, 1)$$

and

$$(4, -3) = a(2, 1) + b(-3, 1).$$

These are routine system of equations problems, so we omit the details. We have

$$P_{C \leftarrow B} = [[(9, 2)]_C \quad [(4, -3)]_C] = \begin{bmatrix} 3 & -1 \\ -1 & -2 \end{bmatrix}$$

2. Consider two ordered bases of $P_1(\mathbb{R})$; $B = \{7 - 4x, 5x\}$ and $C = \{1 - 2x, 2 + x\}$. Find the change-of-basis matrix $P_{C \leftarrow B}$.

We should solve

$$7 - 4x = a(1 - 2x) + b(2 + x) = (a + 2b) + (-2a + b)x$$

and

$$5x = a(1 - 2x) + b(2 + x) = (a + 2b) + (-2a + b)x.$$

The first equation gives us $\begin{matrix} a + 2b & = & 7 \\ -2a + b & = & -4 \end{matrix}$ and the second one gives $\begin{matrix} a + 2b & = & 0 \\ -2a + b & = & 5 \end{matrix}$.

It is a routine exercise again. We have

$$P_{C \leftarrow B} = [[7 - 4x]_C \quad [5x]_C] = \begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$$