QUIZ 9 - MATH 225
SOLUTIONS

1. Find the all eigenvalues and eigenvectors of $A = \begin{bmatrix} 5 & -4 \\ 8 & -7 \end{bmatrix}$.

First, we need to solve the characteristic equation $det(A - \lambda I_2) = 0$. We get

$$det\left(\begin{bmatrix} 5-\lambda & -4 \\ 8 & -7-\lambda \end{bmatrix}\right) = (5-\lambda)(-7-\lambda) + 32 = \lambda^2 + 2\lambda - 3 = (\lambda+3)(\lambda-1) = 0.$$

Thus, we have two eigenvalues $\lambda = -3, 1$.

For $\lambda = -3$, we have the system $\begin{bmatrix} 8 & -4 \\ 8 & -4 \end{bmatrix} \mathbf{v} = \mathbf{0}$. So if $\mathbf{v} = (x, y)$, the equation yields $8x - 4y = 0$ namely $2x = y$. So the eigenvectors are of the form $\begin{bmatrix} x \\ 2x \end{bmatrix}$.

For $\lambda = 1$, we have the system $\begin{bmatrix} 4 & -4 \\ 8 & -8 \end{bmatrix} \mathbf{v} = \mathbf{0}$. So if $\mathbf{v} = (x, y)$, the equation yields $4x - 4y = 0$ namely $x = y$. So the eigenvectors are of the form $\begin{bmatrix} x \\ x \end{bmatrix}$.

Remark. Instead of x, you can use y as a free variable and take $\begin{bmatrix} y/2 \\ y \end{bmatrix}$ and $\begin{bmatrix} y \\ y \end{bmatrix}$ as the eigenvectors.