Name:______________________________ **IU Username:** ________________

Exam 3

In-class

- You will have **50 minutes** to complete this exam.
- This exam is printed double sided.
- This exam will be scanned. Please use only regular pencil or black ink.
- Do **NOT** detach this cover sheet from the exam.
- There is a blank page at the end of the test; it may be detached and used as scratch paper.
- During this exam, you may only use the scratch paper and writing utensils. **No calculators**, cell phones, books, notes or other resources will be permitted.
- **Multiple Choice**: No justification necessary. No partial credit. Fill in the bubble for your answer.
- **Short Answer**: No justification necessary. No partial credit. Write your answer in the box.
- **Free Response**: You must **justify your solution** to receive full credit on a problem. Any of your classmates should be able to understand how you arrived at your solution. Partial credit will be granted for work that demonstrates understanding of key concepts.
- **You can do it!**

Point distribution

Short Answer: 4 problems $\times$ 10 points each = 40 points
Multiple Choice: 2 problems $\times$ 10 points each = 20 points
Free Response: 2 problems 10+30 = 40 points
Total: 100 points

Short Answer

You do **NOT** need to justify your solutions in this section. Simply write your answer in the box. Each problem is worth ten (10) points.

Problem 1. Find the linear approximation to the function $f(x) = \sqrt[3]{x}$ at $a = 1$.

Problem 2. Evaluate $\lim_{x \to 1} \frac{x \sin(x-1)}{2x^2 - x - 1}$.

Problem 3. Find $f(x)$ if

$$f''(x) = 20x^3 + 5, \quad f(1) = 0, \quad f'(1) = 8.$$

Problem 4. Find the absolute maximum and the absolute minimum values of

$$f(x) = x \ln x + 2$$

on $[\frac{1}{10e}, 4]$.

Multiple Choice

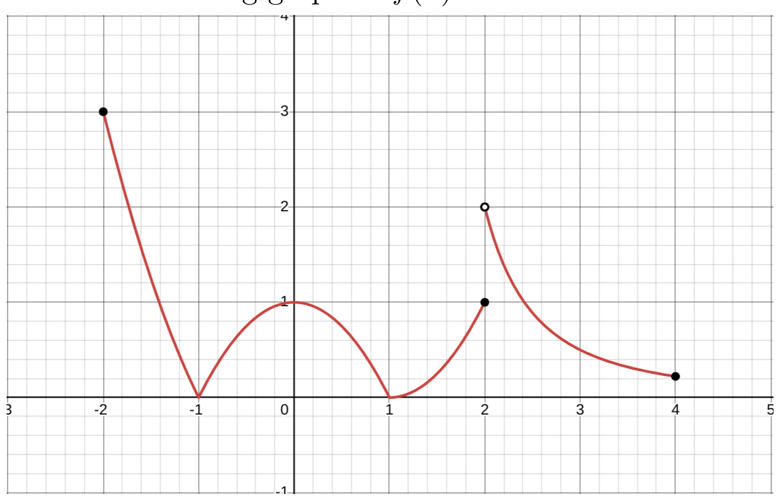
You do **NOT** need to justify your solutions in this section. Simply fill in the circle that corresponds to your answer. Each problem is worth ten (10) points.

Problem 5. Let $f(x) = x + 1$. The mean value theorem for f on the closed interval $[-1, 3]$ asserts that there exists a c in $(-1, 3)$ such that :

Ⓐ $f(c) = \frac{f(3)-f(-1)}{3-(-1)}$ Ⓑ $f(c)$ is a local maximum for f on $[-1, 3]$

Ⓒ $f'(c) = \frac{f(3)-f(-1)}{3-(-1)}$ Ⓓ $f(c) = y$ for $f(-1) \leq y \leq f(3)$

Problem 6. Consider the following graph of $f(x)$:



On which of the following intervals does the function $f(x)$ satisfy all the hypotheses of the Rolle's theorem?

Ⓐ $[-2, 1]$ Ⓑ $[-1, 1]$ Ⓒ $[0, 2]$

Ⓓ $[1, 3]$ Ⓔ $[0, 4]$

Free Response

For each of the questions in this section, you must **provide justification** for your answers.

Problem 7. A box with a square base and no top is to be made from 300 square centimeters of material. Determine the dimensions of the box that will give the largest possible volume.

Problem 8. Consider

$$f(x) = \frac{1-x^2}{1+x^2}, \quad f'(x) = \frac{-4x}{(1+x^2)^2} \quad f''(x) = \frac{-4(-3x^2+1)}{(1+x^2)^3}$$

a. Find the domain of $f(x)$.
b. Find x-intercepts and y-intercepts of $f(x)$.
c. Find the asymptotes of $f(x)$.
d. Find the interval of increasing and decreasing of $f(x)$.
e. Find the local maximum and minimum values of $f(x)$.
f. Find the intervals of concavity, and the points of inflections of $f(x)$.
g. Sketch the graph of $f(x)$.

Continue to solve graph sketch problem.

Scratch Page

Scratch Page